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Electromagnetic Concepts in Mathematical Representation of Physics

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Our paper deals with the use of mathematics when studying the physics of electromagnetism. We have concentrated on common electromagnetic concepts (magnetic field and flux) and their associated mathematical representation and arithmetical tools. Our studies showed that most students do not understand the significant aspects of physical situations. Students have difficulty in using relationships and models specific to magnetic phenomena (the construction of relationships between concepts, and the use of mathematical formalism).

KEY WORDS: Physics; electromagnetism; representations.

INTRODUCTION AND DISCUSSION OF THE ISSUE

During previous studies on the teaching of electric motors (Albe *et al.,* 1999), we asked teachers of the professional and technical baccalaureat[4] about the difficulties students encountered in mastering standard electromagnetic concepts. The concepts of magnetic field, flux, and induction were most frequently mentioned as potential causes of difficulty. Moreover, in the course of our interviews, some teachers had indicated that they felt that the mathematical representation of physical phenomena was a real barrier to understanding.

In this study, we look at the place of mathematics in physics teaching, with special reference to the student's point of view.

Very few studies on the teaching of physics have dealt with electromagnetic concepts (Galili, 1995; Greca and Moreira, 1997; Maarouf and Benyamna, 1997; Viennot and Rainson, 1992).

In a study of students' conceptions of electrical fields, Viennot and Rainson (1992) showed that students prefer a causal reasoning when trying to understand or explain electrical fields: "When *there is no movement of electrical charges and no* manifest effect *of the electrical field, it is* difficult *to accept that the latter* exists. *No* effect: *no* cause."

This type of reasoning when interpreting physical phenomena is linked to the use of mathematics and the characteristic formalism relating two magnitudes: "Magnitude *X, contained in the algebraic expression of magnitude G = f (X), is interpreted as an exclusive cause of the phenomenon associated with magnitude* G."

Maarouf and Benyamna (1997) also showed that linear causal representations (Rozier, 1987; Tiberghien and Delacote, 1976) underpin students' explanations of magnetic phenomena.

Elsewhere, Galili (1995) demonstrated that university students misunderstand the nature of the interactions and conversions between work and energy in the presence of a magnetic field. He suggests that these difficulties are due to the influence of the teaching of mechanics on students' reasoning in the field of electromagnetism.

Greca and Moreira (1997) state that most students in the second year of Engineering School demonstrate poor organization of knowledge. Their mental representation of the magnetic field is a propositional representation (a definition or a formula) which they manipulate routinely to resolve the traditional problems of electromagnetism.

The same authors, moreover, put forward the hypothesis that the emphasis placed on the mathematical aspects of field lines impedes physical under-standing of the magnetic field.

For Maarouf and Benyamna (1997), high-school pupils (between the ages of 13 and 20) in Morocco have enormous difficulty in manipulating different physical concepts (magnetic interaction, force, speed, current, field) simultaneously and in choosing those that are suitable for explaining a given problem. They add, "these difficulties *are compounded by those that result from the use of a high level, mathematical* formalism."

Moreover, several authors have stated difficulties in understanding the mathematics in other physical concepts: in classical and wave mechanics (Haertel, 2000), special relativity (Ireson, 1996, 2000), quantum theory (Hobson, 2000).

However, as Henry (1996) says, "In *the physics class, the routine use of at least some mathematics,* essentially, *arithmetic, algebra or analytical tools, would seem to be* inevitable." With this in mind, we observed the way in which university students use mathematics when studying electromagnetism.

METHODOLOGY

We can approach the organization of knowledge in different academic disciplines either from the point of view of the related teaching skills or from that of the student. The latter can be examined using problem-solving situations, even if these have no pedagogical objective and it is this perspective that we have discussed here.

Our chosen context is the teaching of physics in which mathematics intervenes as a tool. In the first instance, we dealt with the university level and questioned undergraduates and trainee teachers in physical sciences. The first results of our study concern certain physical and mathematical aspects of magnetic field and flux.

First of all, we interviewed 50 students enrolled in the teacher training program, before their course on electromagnetism. We asked them to define flux and to describe its physical significance. We asked the students to work in pairs when answering the questions. We recorded, transcribed, and analyzed all of their discussions.

We used the results obtained to draw up a multiple-choice questionnaire which we put to 64 physical science undergraduates at the beginning of their degree course during practical work on the teaching of science. The students we questioned did not have a course in electromagnetism, but had taken one the year before.

Our objective for choosing these students was to examine the elements that can be activated by the participants after a quite long period of time without courses of electromagnetism.

The questionnaire dealt with the physical definition of flux and the mathematical formulae used to calculate it. We drew up a second questionnaire composed, on the one hand, of open-ended questions (which were handed out first) and on the other of multiple-choice questions (which were handed out second.) It was given to the same undergraduates. The questionnaire dealt with the characteristics of the magnetic field and its graphic representation.

The results were coded and analyzed using "Le Sphinx," a software for statistical analysis. Even though the sample was limited to 64 people, we decided to present all the results in percentage form to make the results easier to read and analyze.

RESULTS

Teacher Training Students

For the teacher training students, we categorized their answers by classifying the phrases they used in their discussions. Their physical definition of magnetic flux shows that they believe that flux corresponds to

- the transfer of a magnitude (generally the magnetic field) across a surface area (40% of students),

  "It's *the magnetic field which moves across a surface* area"
- a "flow" of magnetic field (28% of students),

  "The quantity of magnetic field that crosses a surface area for a given unit of time"
- the possibility of moving a quantity of various elements across the surface area (12% of students),

  "The movement of a magnetic field across a surface."

  "A current that crosses a surface unit."

If you agree with any of the following statements, tick the appropriate box. If you prefer a different definition, please write it in the space provided.

- ☐ *Magnetic flux is the amount of magnetic field which crosses a surface area*
- ☐ *Magnetic flux is the amount of magnetic field that crosses a surface area for a given unit of time.*
- ☐ *The magnetic flux across a surface area is linked to the number of field lines that cross it .*
- ☐ *The magnetic flux is linked to the movement of the magnetic field across the surface area .*
- ☐ *Magnetic flux....*

Fig. 1. Question on the physical definition of flux. Elements of correct answers. The magnetic flux across a surface area is linked to the number of field lines that cross it.

– nothing in particular (20% of students gave no reply or attempted, unsuccessfully, to find a mathematical solution).

"For me, a magnetic flux, it's an integral "

" What *is B, Idl, ?* "...

"$B = \mu_0 L$ "

"I don't know, actually I begin with Maxwell equations, div B = 0."

It appears from the study, carried out with teacher trainees preparing for the Capes[5] examination in physical sciences, that the first approach used to define the notion of flux is essentially mathematical: 76% of the students, when asked for a definition, first gave a formula for calculating flux; 92% of them gave the traditional formula $\Phi = \oint \vec{B}\vec{n}\, ds$ in their discussions.

Undergraduate Students

The undergraduates were asked to answer a more structured question to give a physical definition of magnetic flux (see Fig. 1). The answers are given in Table I.

These results demonstrate the importance of the answers "amount of magnetic field crossing a surface area by unit of time (flow)" (50% of students) and "amount of magnetic field crossing a surface area" (36%).

Table I. Physical Definition of Magnetic Flux

Answers	Number of students	Percentage of total
Amount of B across S	23	36%
Amount of B across S/t	32	50%
Flux linked to the number of field lines	13	20%
Movement of B	14	22%
No reply	2	3%

Concerning the determination and use of the formulae for calculating magnetic flux, the undergraduate students had to choose at least one of the proposals in Fig. 2. 92% of them chose the traditional formula. Only 22% gave both this reply and a second from among its direct derivatives if the field is uniform: $\phi = \vec{B} \cdot \vec{S}$ or $\phi = BS \cos\theta$

Another question for the undergraduate students concerned the factors affecting flux (see Fig. 3). The percentage of correct answers was as follows:

– 64% for the increase in flux with surface area
– 58% for the increase in flux with magnetic field
– 36% for the reduction in flux with the angle between the normal to the surface and the field.

Moreover, we dealt with the two ways of representing the magnetic field (vectors and field lines) and then with their use in several simple cases.

The first question we asked ("A magnetic field is frequently represented by a vector. Why?") asks students why a vector was chosen to represent a magnetic field. Not a single student gave the reason: they all responded by immediately introducing the characteristics of vectors; 39% gave the three characteristics (intensity, direction, field orientation), 56% gave direction and field orientation, and 73% gave only direction or field orientation. None of the students explained the reason for choosing a vectorial representation for the magnetic field.

Which of the following formulae may be used to calculate flux? You may choose more than one option.

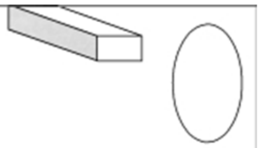
☐ $\phi = \vec{B}.\vec{S}$ ☐ $\phi = B\,S\,tg\theta$ ☐ $\phi = \frac{dB}{dt} S$

☐ $\Phi = \oint \vec{B}\vec{n}\,ds$ ☐ $\phi = B\frac{dS}{dt}$ ☐ $\phi = B\,S\cos\theta$

☐ $\phi = \frac{B}{S}$

Fig. 2. Question on the formulae relating to magnetic flux. Elements of correct answers: $\Phi = \oint \vec{B}\vec{n}\,ds$, $\phi = \vec{B}\cdot\vec{S}$, $\phi = BS\cos\theta$.

You have a magnet and a coil at your disposal



a) If you increase the surface area of the coil, the flux	*diminishes*	☐ *Yes*	☐ *No*
	increases	☐ *Yes*	☐ *No*
	stays the same	☐ *Yes*	☐ *No*
b) If you move the magnet towards the coil , the flux	*diminishes*	☐ *Yes*	☐ *No*
	increases	☐ *Yes*	☐ *No*
	stays the same	☐ *Yes*	☐ *No*
c) If you tilt the coil, the flux	*diminishes*	☐ *Yes*	☐ *No*
	increases	☐ *Yes*	☐ *No*
	stays the same	☐ *Yes*	☐ *No*

Fig. 3. Question on the factors affecting flux. Elements of correct answers: (a) the flux increases, (b) the flux increases, (c) the flux diminishes.

As far as field lines are concerned ("*A magnetic field can be represented using field lines. How is this done?,*"), 61% of students failed to reply or gave in-correct answers. 26% stated that the field lines are at a tangent to the magnetic field (without defining its orientation). Only two students set out all of the characteristics of field lines: direction, field orientation, and density in relation to the direction, the orientation, and the intensity of the magnetic field.

The next question (see Fig. 4) set out two equivalent vectorial representations of a uniform field and the students were asked to comment on their relevance. 54% of the students considered the two representations to be equivalent. The other replies show an almost homogeneous spread.

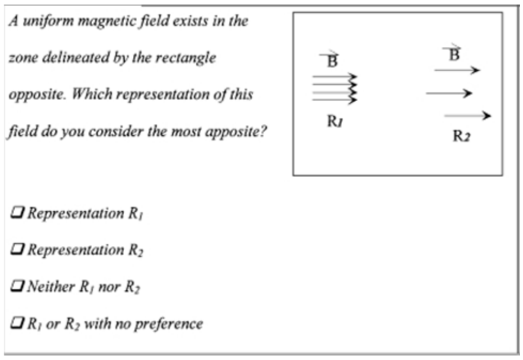


Fig. 4. Question concerning the representations of a uniform field. Elements of correct answer: R1 or R2 with no preference.

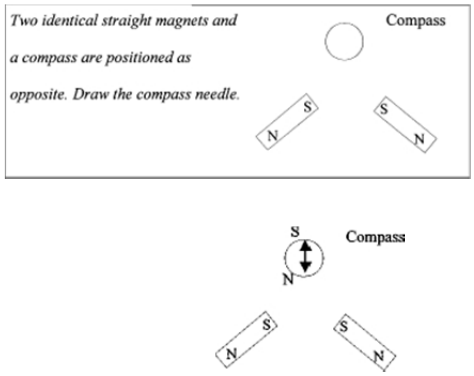


Fig. 5. (a) Orientation of a compass influenced by two magnets. (b) Elements of correct answer.

When asked to predict the orientation of a compass needle subjected to the effects of two identical magnets (see Fig. 5), 43% of students gave a correct answer, whereas 42% showed the direction of the needle correctly without defining the orientation.

Finally, we asked the students to draw, where it exists, the field alongside various objects: a straight magnet, a u-shaped magnet, a compass, a coil with a current passing through it, a glass rod, and a horseshoe. As well as the classic elements that must be included when studying students' understanding, we introduced the glass rod for its relation-ship with electrostatics and the horseshoe for its closeness in shape, matter, and description to the u-shaped magnet. An outline of the results is given in Fig. 6.

We can see that there were many blank answers and that, on the other hand, there were few answers that contain indications of both orientation and direction of the field. Intensity was almost never taken into account, whether by the density of the field lines or the vectorial norm.

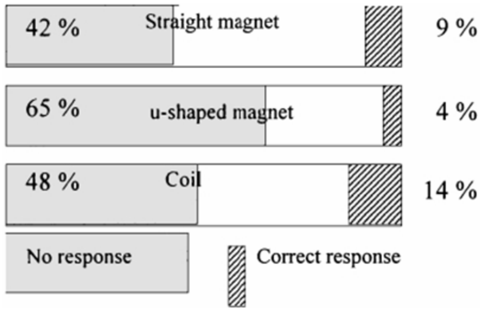


Fig. 6. Representation of the magnetic field produced

DISCUSSION

Giving a Physical Definition of Magnetic Flux

The teacher training students appear to have constructed their explanations on the basis of

– • common definitions of the word "flux" (possibly as though it was a physical substance),
– • vocabulary employed in physics courses (flux "across" a surface area), and
– • the association between the vectorial representation of the field and the idea of movement (shown by the arrow).

Most of the undergraduates confined themselves to the same items and did not propose new ones.

The physical definition of magnetic flux appears to be confused for both groups. A partial explanation could be that flux is often introduced in a strictly mathematical manner and that this concept only acquires physical legitimacy later with the study of induction phenomena. Explanations for the reasons and sources of those difficulties may also be related to unexplained breaks with previous concepts, missing conceptual explanations, emphasizing on solving few kinds of standard (usually quantitative) problems.

Determining and Using the Formula for Calculating Magnetic Flux

The majority of students from both groups recognized at least one of the formulae for calculating flux. These do not involve time. However, when a verbal explanation of flux was required, half of the undergraduate students and more than a quarter of the teacher training students chose the definition "Amount of magnetic field across the surface area for a unit of time," which then brings time into the equation (see Section 3). We could therefore say that for these students there is lack of clarity between the verbal explanation of the physics phenomena and the interpretation of the mathematics formula.

Table II. Number of Correct Answers to Questions on the Calculation of Flux

Number of correct answers out of four possible	Number of students	Percentage of total
0	1	2%
1	8	12%
2	20	31%
3	27	42%

the following comments only apply to the under-graduate students. Nearly all (94%) referred to the surface area and the magnetic field correctly in the formula for calculating flux. However, only 64% of them took full account of the increase in flux with the increase in surface area. This percentage decreases (58%) when it was a question of variations in the field. It should however be noted that, in order to answer the question correctly, it is also necessary to know that when one brings a magnet close to a point, the field at that point is increased. Equally, as far as the link between the variation in flux and the tilt of the coil is concerned, the students did not know how to interpret the notion of scalar product in physical terms: in fact, only 36% gave the correct answer, whereas 94% gave at least one correct formula for calculating flux

This should, moreover, be interpreted while keeping in mind the fact that of those students who gave $\phi = \vec{B} \cdot \vec{S}$, as the formula for flux, only half also gave $\phi = BS \cos\theta$, which is the straightforward, common mathematical explanation.

We may therefore conclude that the traditional formula, which is widely quoted, is not equally effective in simple cases, as Table II also shows from the number of correct answers to the following questions: formula for flux, variations in flux as a function of surface area, field, and tilt. We can see that only 13% of students knew how to apply the formula correctly, while taking the variations in the different factors into account.

Other studies have demonstrated the same phenomenon. "Students who do cope with the mathematics course are still unable to apply it in context." (Gill, 1999). For instance, Viard (1995) asked undergraduate students to solve an elementary electrokinetic problem. The students were able to quote the formulae (the calculation of equivalent resistances) which allow the evolution of the circuit to be predicted without, however, managing to make a correct prediction. Equally, Alibert et al. (1988) studied the cooperation between mathematics and physics at university level on the theme of differentials. They showed that in both mathematics and physics, the students systematically prefer automatic, algorithmic procedures.

This type of reasoning is preferred overwhelmingly to the detriment of reflection on the role and status of differentials and of differential procedures, in mathematics and in physics. Questions could therefore be asked about the role of the institutions that condition students' relationship to knowledge. Indeed, as the same authors have noted,

> *In the teaching of the two disciplines, there is a sort of consensus which encourages and develops algorithmic procedures rather than more conceptual aspects ... In fact, students see no need for such conceptual considerations when solving the problems they are usually given. They use a few linguistic markers, which they link to procedures whose conceptual basis has been forgotten.*

Representing a Magnetic Field

It is apparent that the students did not make a formal link between the physical concept that makes up the magnetic field and its representational modes. Drawing a vector or field lines seems to be a meaningless activity and, what is more, badly done. The representations handed were of poor quality. Thus, although the students generally mentioned field direction in their replies, they often omitted the orientation and nearly always its intensity. The poles rarely appeared on drawings of the magnets and the compass needle, the field lines had no direction, their density was rarely mentioned, and the norm of the vectors represented did not change.

These comments reflect an idea developed by Amigues and Caillot (1990) concerning electrical circuit diagrams and which states that they become autonomous entities divorced from the concepts from which they derive. "In physics teaching, graphics play a decisive representational role whether for presenting conceptual frameworks or for accessing physical models or research paradigms." Faraday introduced the concept of field through lines of force. The only measurable quantity that he introduced was the relation between the number of lines of force and the magnitude of the induced force (Nersessian, 1992). The students in our sample saw a field line as at best a tangent to the magnetic field.

Moreover, one would suppose that the questions set out in Figs. 4 and 5, which deal with the representation of a uniform field and the composition of two fields, would be answered correctly more easily by those who understand why the vector is a satisfactory representation of the field. On the other hand, one would also suppose that the students who answered the questions correctly were capable of quoting the three vectorial field characteristics. This was not at all the case; of the 25 students who gave the three characteristics and the 28 who managed to compose the fields (Fig. 5), only 14 did both. Similarly, of the 25 students who gave the three vectorial field characteristics and the 35 who recognized the identical character of the representations (Fig. 4), only 11 did both. Finally, only six students answered all the three questions correctly. We found a similar phenomenon when we compared the results concerning the vectorial characteristics associated with the field and the representations of the field produced by a straight magnet and a coil. Of the 25 students who gave the three characteristics, only five also managed to draw the field produced by a straight magnet. Similarly, of the 25 students who gave the three vectorial field characteristics, only three also managed to draw the field produced by a coil. Once again, we can see that the students did not connect different pieces of knowledge and did not apply them to similar situations.

In the same way, we can note that the use of one representation does not always lead to a correct use of other representations. We observed that 58% of the students used correctly the formula for calculating flux with the increase of the magnetic field produced by a straight magnet, but only 9% managed to draw correctly the field produced by a straight magnet. This again supports the idea that the students' knowledge is fragmented.

Further evidence may be found in Table III, which draws up the number of correct replies to the six questions we have mentioned: vectorial representation of the field, representation of the field using field lines, representation of a uniform field, composition of two fields, representation of the field of a straight magnet, representation of the field of a coil. These questions cover the fundamental elements required in the representation of a magnetic field. We can see that no student gave five or six correct answers out of the six possible; only about 1/5 of them gave three or more answers. The knowledge brought into play was therefore limited and isolated: the students had a poor grasp of the complete set of characteristics involved in representing a magnetic field: vectors and field lines.

Table III. Number of Correct Replies to 6 Questions on the Representation of Magnetic Fields

Number of correct answers out of six possible	Number of students	Percentage of the total
0	9	14%
1	19	30%
2	24	37%
3	11	17%
4	1	2%

CONCLUSION

In this paper, we have dealt with the use of mathematics in physics teaching, concentrating on fundamental electromagnetic concepts: magnetic field and flux.

Our study demonstrates that students have difficulty in using relationships and models which are specific to magnetic phenomena (construction of relationships between concepts, use of mathematical formalism).

It seems that the elements of knowledge brought into play constitute "islands" which most of the students do not connect fully, this being an analogy for all the links that make up the basic concepts of electromagnetism. The use of physical knowledge is fragmented and the students are not able to bring together different elements of knowledge. This indicates a lack of conceptual grasp of the basic physics of electromagnetism.

Furthermore, we observed that most students have problems in associating mathematical formalism (vector, integral calculus) with physical descriptions of magnetic field and flux. The students also have difficulty in using this formalism in elementary situations. Mathematics is the starting point, but our observations would indicate that the link with mathematics is almost entirely procedural.

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